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Standard Distributions

Discrete Distributions	Continuous Distributions
* Binomial Distribution	* Uniform Distribution
* Poisson Distribution	* Exponential Distribution
	* Normal Distribution.

Binomial Distribution:

A random variable x is said to follow Binomial Distribution if it assumes only non-negative values and it's probability mass function is given by

$$P[x=x] = nC_x \, p^x q^{n-x}, \quad x = 0, 1, 2 \ldots n$$

and $q = 1-p$

Notation

$X \sim B(n,p)$

n and p are parameters of Binomial Distribution.

Remarks

$$\sum P(x=x) = \sum_{x=0}^{n} nC_x \, p^x q^{n-x} = (q+p)^n = 1.$$

If N is the total frequency, expected frequencies of 1, 2, ... n success are the successive terms of $N(p+q)^n$

Derive MGF, Mean and Variance of Binomial Distribution

Probability Mass function of Binomial Distribution is

$$P[X=x] = \sum_{x=0}^{n} nC_x \, p^x q^{n-x}, \quad x = 0, 1, 2 \ldots n.$$

Moment Generating Function

$$M_X(t) = E\left[e^{tx}\right] = \sum_{x=0}^{n} e^{tx} P(x).$$

$$= \sum_{x=0}^{n} e^{tx} \, nC_x \, p^x q^{n-x}.$$

$$= \sum_{x=0}^{n} nC_x (pe^t)^x q^{n-x}.$$

$$= nC_0 (pe^t)^0 q^n + nC_1 (pe^t)^1 q^{n-1} + \ldots + nC_n (pe^t)^n q^0$$

$$\boxed{M_X(t) = (pe^t + q)^n} \quad (\because \text{by Binomial expansion}).$$

$$\text{Mean} = \left[M_X'(t)\right]_{t=0}$$

$$= \frac{d}{dt}(pe^t + q)^n \Big|_{t=0}.$$

$$= \left[n(pe^t + q)^{n-1} pe^t\right]_{t=0}.$$

$$= n(p+q)^{n-1} p.$$

$$\left[\because q + p = 1 \Rightarrow (p+q)^{n-1} = 1^{n-1} = 1\right]$$

$$\boxed{\text{Mean} = np.}$$

$$E[x^2] = \left[M_x''(t)\right]_{t=0}$$

$$= \left[\frac{d}{dt}\left\{npe^t\,(pe^t+q)^{n-1}\right\}\right]_{t=0}$$

$$= np\left[\frac{d}{dt}\left[e^t\,(pe^t+q)^{n-1}\right]\right]_{t=0}$$

$$= np\left\{e^t\,(n-1)\left[pe^t+q\right]^{n-2}pe^t + (pe^t+q)^{n-1}e^t\right\}_{t=0}$$

$$= np\left\{(n-1)(1)\,p+1\right\}.$$

$$= np\left\{np-p+1\right\} = n^2p^2 - np^2 + np.$$

$\therefore$ Variance $= E[x^2] - (E(x))^2$

$$= n^2p^2 - n^2p + np - (np)^2.$$

$$= \cancel{n^2p^2} - n^2p + np - \cancel{n^2p^2}$$

$$= np(1-p) = npq \qquad (\because 1-p = q)$$

$$\boxed{\text{Variance} = npq}$$

Remark

For Binomial Distribution, Mean > Variance.

Properties of Binomial Distribution

* The experiment consists of 'n' repeated trials
* The repeated trials are independent
* The probability of success denoted by 'p' remains constant from trial to trial

Remarks:

* $P[\text{all success}] = p^n$
* $P[\text{no success}] = q^n$
* $P[\text{atleast one success}] = 1-q^n$

For better results use Binomial distribution when $n \leq 30$, $p < 0.5$.

Additive Property of Binomial Distribution

By Additive property of MGF. we're

$$M_{x+y}(t) = M_x(t) M_y(t).$$

Let $M_x(t)$ and $M_y(t)$ be the MGF of the Binomial variates x and y with parameters (n_1, p) and (n_2, p) then

$$M_{x+y}(t) = (pe^t+q)^{n_1} (pe^t+q)^{n_2}.$$

$$M_{x+y}(t) = (pe^t+q)^{n_1+n_2}.$$